

**From:** [Leinenbach, Peter](#)  
**To:** [FRUEH Terry](#)  
**Cc:** [Kubo, Teresa](#); [Henning, Alan](#)  
**Subject:** RE:  
**Date:** Wednesday, April 16, 2014 8:08:24 AM  
**Attachments:** [Shadow Length Memorandum.pdf](#)

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Terry –

I have attached the memo which you requested for below.

Peter

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**From:** FRUEH Terry [mailto:[terry.frueh@state.or.us](mailto:terry.frueh@state.or.us)]

**Sent:** Tuesday, April 15, 2014 1:33 PM

**To:** Leinenbach, Peter

**Subject:**

Peter,

Could you please send me a copy of:

Potential Shadow Length Associated with Riparian Vegetation

Leinenbach, P, 2011. Technical analysis associated with this project to assess the potential shadow

length associated with Riparian vegetation

It will be helpful to have that analysis for thinking about our model.

Thanks,

Terry

W. Terry Frueh

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## Technical Memorandum

December 2011

**From:** Peter Leinenbach, USEPA

**To:** Personal File

**Subject:** Assess the potential shadow length associated with riparian vegetation

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**Synopsis:** Results indicate that a tree located on a flat hillslope along the stream within a distance of its height can be influential on shade production (i.e., the shadow length associated with the tree is long enough to reach the stream), and ultimately on stream temperature during the summer period (July/August). However, there are commonly occurring situations which trees outside of this distance can contribute to shade production (For example, a 100 foot tall tree located on a hillslope of 20 degrees can cast a 169 foot long shadow at 4 PM during the late summer.).

### **Riparian Stand and Harvest Conditions:**

Sites: Sensitivity analysis of shadow length associated with vegetation at a 45.7°N Latitude

Stand Conditions: Variable tree height

Time line: Estimates during the spring, summer, and fall period.

### **Summary of Results:**

The distance of a shadow cast by a tree can be estimated by the following trigonometric equation<sup>1</sup>:

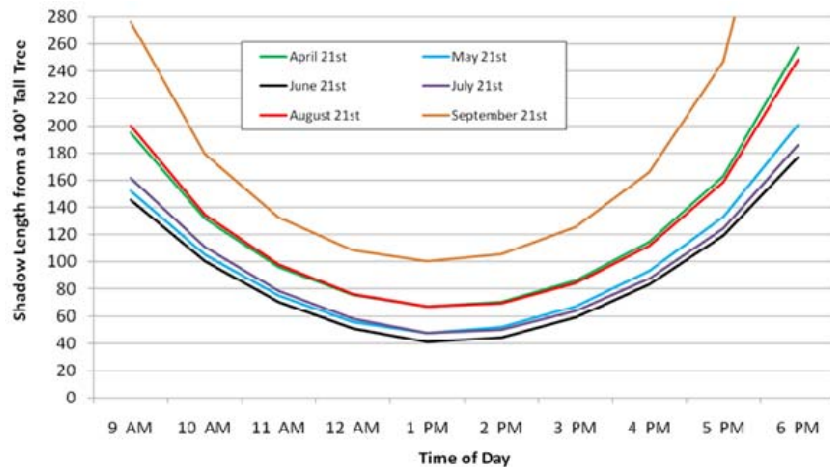
$$\text{Shadow Length} = \frac{\text{Tree Height} * \cos(\text{Hillslope Angle})}{\tan(\text{Sun Angle} - \text{Hillslope Angle})} - \text{Tree Height} * \sin(\text{Hillslope Angle})$$

Solving this equation provides insight into the distance from a stream a tree could potentially provide stream shade. The tree will not have any effect of stream shade production when it is located further away from the stream than the calculated shadow length. The figure below shows that the shadow distance associated with a 100' tall tree varies throughout the course of the day, along with the season<sup>2</sup>. The shadow distance increases as the sun is lower in the sky during the mid morning (9 am to 11 am) and mid afternoon (2 pm to 4 pm) periods. The figure also indicates that shadow lengths are longer during late spring and late summer, than during the summer equinox.

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<sup>1</sup> See Attachment A below for the derivation of this equation.

<sup>2</sup> The "Altitude of the Sun" reference location associated with analysis was within the Tillamook Forest and the model used to determine the "Altitude of the Sun" (i.e., SolRad) was obtained from Washington Ecology's TMDL model webpage - <http://www.ecy.wa.gov/programs/eap/models.html>



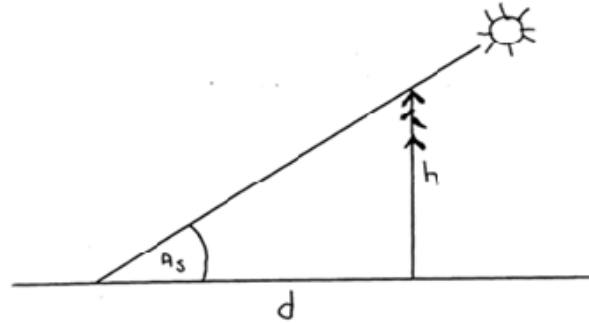
Stream temperatures are often at a maximum during the July to August period and therefore stream shade is particularly important at this time<sup>3</sup>. The table below presents the average shade length associated with riparian vegetation during these summer months. On a flat stream bank, the shadow length can equal the height of the tree in the afternoon, when stream temperatures are often at their daily maximum and potential solar heat loading is still high (i.e., 4 pm conditions). The table below also shows that the shadow length increases for vegetation located on sloped stream banks.

Average July 21 <sup>st</sup> and August 21 <sup>st</sup> shadow length (feet) associated various tree height conditions.										
Height of Tree	9 am	10 am	11 am	12 pm	1 pm	2 pm	3 pm	4 pm	5 pm	6 pm
Flat Hillslope										
20	36	25	18	13	11	12	15	20	28	43
40	72	49	35	27	23	24	30	40	57	87
60	108	74	53	40	34	36	44	60	85	130
80	145	98	71	54	46	48	59	80	113	174
100	181	123	88	67	57	60	74	100	142	217
120	217	147	106	81	69	72	89	120	170	260
140	253	172	124	94	80	84	104	140	198	304
20 Degree Hillslope										
20	120	48	28	19	16	16	22	34	64	334
40	240	96	56	38	31	33	44	68	129	668
60	360	143	84	57	47	49	65	101	193	1002
80	480	191	112	76	62	66	87	135	257	1336
100	599	239	140	95	78	82	109	169	321	1669
120	719	287	167	115	94	99	131	203	386	2003
140	839	335	195	134	109	115	152	237	450	2337

<sup>3</sup> July and August (and sometimes September) conditions are often associated with low stream flows, long days, and warm air temperatures, which can result in high stream temperatures. Therefore, rivers/streams often have lower assimilative capacity for the addition of heat loads.

## Attachment A – Estimating Shadow Distances

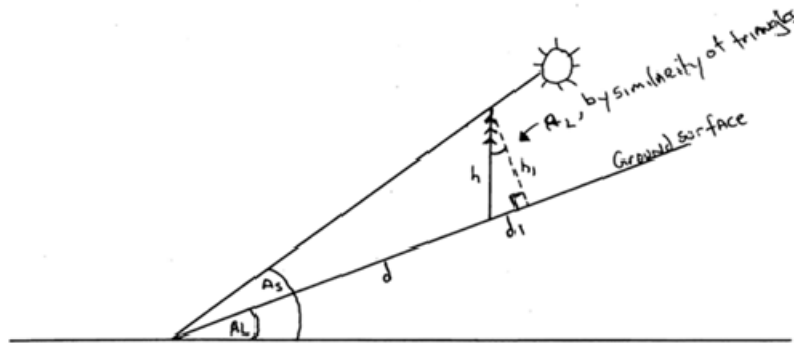
Case 1: Ground has Zero Slope



$A_s$  = sun angle,  $h$  = tree height, and  $d$  = shadow distance

$$\tan(A_s) = \frac{h}{d} \Rightarrow d = \frac{h}{\tan(A_s)}$$

Case 2: Ground is sloped, with a slope angle =  $A_L$  and assume that the tree grows vertically



$A_s$  = sun angle above the horizon, not the ground surface,  $h_1$  = height of the line drawn from the tree tip, perpendicular to the ground, and  $d_1$  = distance from interception of that line with the ground, to the base of the tree.

Using the same argument as in Case 1,

$$\tan(A_s - A_L) = \frac{h_1}{(d_1 + d)}$$

Solve this for d, the shadow distance:

$$d = \frac{h}{\tan(A_S - A_L)} - d_1$$

Since,

$$h_1 = h * \cos(A_L) \text{ and } d_1 = h * \sin(A_L)$$

Thus,

$$d = \frac{h * \cos(A_L)}{\tan(A_S - A_L)} - h * \sin(A_L)$$

In other words,

$$\text{Shadow Length} = \frac{\text{Tree Height} * \cos(\text{Hillslope Angle})}{\tan(\text{Sun Angle} - \text{Hillslope Angle})} - \text{Tree Height} * \sin(\text{Hillslope Angle})$$

Note: When  $A_L = 0$  (flat ground), this equation reduces to Case 1, because  $\sin(0) = 0$ , and  $\cos(0) = 1$